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WiSe 2016/17

1. Problem Set
Advanced Statistical Physics

Due Date: Thursday, October 27, 10am

Please indicate your name and the number of your group on the first page!

Problem 1 *Van-der-Waals & Dieterici* **12 Points**

Consider a van-der-Waals gas with the equation of state

$$\left(p + \frac{a}{v^2}\right)(v-b) = k_\mathrm{B}T$$

and constant heat capacity C_V.

a) Calculate the entropy $S(T,V)$, and justify your choice of the path for the path integral.

b) Calculate the internal energy $E(T,V)$ and the free energy $F(T,V)$.

Consider the Dieterici equation of state

$$p(v-b) = k_\mathrm{B}T \exp\left(-\frac{a}{k_\mathrm{B}Tv}\right)$$

c) Determine the critical point p_c, T_c, v_c of the Dieterici gas.

Problem 2 *The characteristic function* **8 Points**

Let $w(x)$ be a probability density. One calls $\varphi(k) = \int_{-\infty}^{\infty} dx w(x) e^{ikx}$ the *characteristic function* of this distribution.

a) Show the following relation between the n-th derivative of the characteristic function and the n-th moment of the distribution $\langle x^n \rangle = \int dx x^n w(x)$

$$\langle x^n \rangle = (-i)^n \frac{d^n\varphi}{dk^n}(0).$$

b) Show that the characteristic function $\varphi_G(k)$ of a Gaussian distribution

$$w_G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

solves the following differential equation

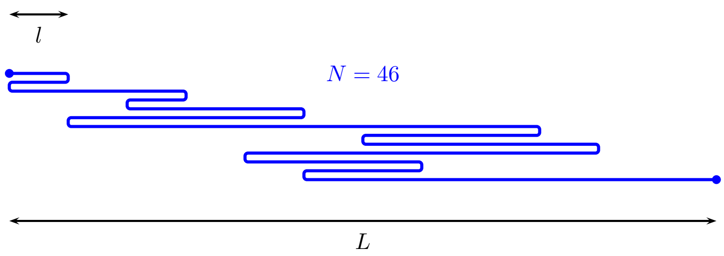
$$\frac{d\varphi_G}{dk}(k) = (i\mu - \sigma^2 k)\varphi_G(k).$$

To this end, rewrite $\frac{d\varphi_G}{dk}(k)$ in terms of $\varphi_G(k)$

c) Show that this differential equation is solved by the ansatz $\varphi_G(k) = \exp(ik\mu)\exp(-\sigma^2 k^2/2)$.

Problem 3 *Model of a rubber band, Part I* **10 Points**

Consider an one-dimensional chain made up of N segments of length l, each of which can point with equal probability to the right or to the left (relative to an arbitrarily chosen starting point, respectively, running direction along the chain). The number of segments pointing to the right be n_+, and of those pointing to the left, n_-.



a) Calculate the number $\Omega_N(L)$ of possible configurations of length L (distance between start and end point) L.

b) Determine the probability $P_N(L)$ of obtaining a configuration of length L.

c) Calculate the entropy of the chain as a function of L. Consider the approximation for large N using Stirling's formula.