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WiSe 2016/17

4. Problem Set
Advanced Statistical Physics

Due Date: Thursday, November 17, 10am

Please indicate your name and the number of your group on the first page!

Problem 10 *Domain Walls in low-dimensional systems* **8 Punkte**

We want to analyze the stability of the ordered ground state of the Ising model against thermal fluctuations in one and two dimensions.

a) Imagine we flip a large number of spins on one end of a one-dimensional Ising chain. Determine the energy and entropy change due to the appearance of the domain "wall" between the up-spin and the down-spin region. For what temperatures does the resulting free energy favor domain walls? What does this mean for the ordered phase?

b) Now consider a droplet of down-spins in a 2d Ising model with only up-spins. Again estimate the energy cost and entropy gain due to the domain wall. What does the temperature dependence of the corresponding free energy tell you about the stability of a ferromagnetic phase at finite temperature?

Problem 11 *Domain Walls in Landau Theory* **12 Punkte**

Consider the Ginzburg-Landau free energy functional

$$\mathcal{F}[m(\boldsymbol{r})] = \int d^3r \left[Atm^2(\boldsymbol{r}) + Bm^4(\boldsymbol{r}) + \kappa\big(\nabla m(\boldsymbol{r})\big)^2 \right]$$

Assume we are in the low temperature phase, $t < 0$, and there is a domain wall at the $(x = 0)$-plane. That is, the system has, say, magnetization m_0 for $x \to +\infty$ and magnetization $-m_0$ for $x \to -\infty$.

a) In analytical mechanics, the stationarity of the action functional led to the Euler-Lagrange Equations. Show how the analogous procedure can be used to obtain equations for the domain wall profile $m(x)$ and solve these equations to show that

$$m(x) = m_0 \tanh(x/w)$$

What determines the domain width w?

b) Calculate the free energy of the domain wall as the difference between the free energy for the profile derived above and a flat profile $m(x) = m_0$.

Problem 12 *Landau Theory for Ferroelectricity* **10 Punkte**

Consider a ferroelectric material that shows spontaneous polarization at a transition temperature $T = T_c$ (and zero electric field $\boldsymbol{E} = \boldsymbol{0}$). To sixth order in the polarization $\boldsymbol{P} = P\boldsymbol{e}_z$ we can write the Landau free energy as

$$F(\boldsymbol{P}, T) = F_0(T) + \frac{1}{2}a(T)P^2 + \frac{1}{4}b(T)P^4 + \frac{1}{6}c(T)P^6$$

a) Discuss what sign a, b, and c must have that F fulfills the following conditions

   i. Close to T_c, F shall have three minima
   ii. Above T_c, the absolute minimum shall be at $P = 0$
   iii. F shall be continuous in T_c and
   iv. stable for $P \to \infty$.

   Calculate the spontaneous polarization as a function of a, b, c

b) Add a term $-\boldsymbol{E} \cdot \boldsymbol{P}$, coupling to the external field $\boldsymbol{E}$ and calculate the dielectric susceptibility $\chi = \partial \boldsymbol{P} / \partial \boldsymbol{E}$ close to T_c

c) Consider the special case that only the coefficient $a(T) = At$ ($A > 0$) is temperature dependent. Determine T_c in this case and discuss the behavior of the susceptibility.