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WiSe 2016/17

5. Problem Set
Advanced Statistical Physics

Due Date: Thursday, November 24, 10am

Please indicate your name and the number of your group on the first page!

Problem 13 *A Tricritical Point* **10 Punkte**

The following Ginzburg-Landau-functional holds close to a so called *tricritical point*

$$\mathcal{F}[\phi] = \int d^3r \left[a\phi^2 + c\phi^6 + \kappa(\nabla\phi)^2 - H\phi\right]$$

with no fourth order term, $c > 0$ (why?), and the ususal form $a(T) = At$. Determine the homogeneous stationary solution $\overline{\phi}$ at zero external field ($H = 0$) and the critical exponents.

Problem 14 *A Stripe Phase* **10 Punkte**

Consider a Landau free energy

$$\mathcal{F}[M] = \int_0^L dx \left[AtM^2(x) + BM^4(x) + \kappa\big(\partial_x M(x)\big)^2 + \sigma\big(\partial_x^2 M(x)\big)^2\right]$$

with $A, B > 0$ as usual but κ may be either positive or negative. Here, with $\sigma > 0$ curvature, $\partial_x^2 M$, is penalized and not gradients.

Write $\mathcal{F}$ in terms of the Fourier components $\tilde{M}_n$ corresponding to the wave number $q_n = 2\pi n/L$. Minimize with respect to $\tilde{M}_n$ and n and show that there are three phases near $t = 0$: (i) a paramagnetic phase $\tilde{M}_n = 0$, (ii) a ferromagnetic phase $\tilde{M}_0 \neq 0$, and (iii) a spatially modulated phase $\tilde{M}_n \neq 0$ for some $n \neq 0$. Sketch the phase diagram in the $t - \kappa$ plane and discuss whether the transitions between the phases are continuous or first order.

Problem 15 *Gaussian Integrals* **10 Punkte**

Consider a real symmetric $n \times n$ matrix A

a) Prove that

$$\langle x_a x_b\rangle := \frac{\int d^n x x_a x_b e^{-\frac{1}{2}A_{ij}x_i x_j}}{\int d^n x e^{-\frac{1}{2}A_{ij}x_i x_j}} = A^{-1}_{ab}$$

where the Einstein summation convention is employed.

b) Prove

$$\langle x_a x_b x_c x_d\rangle = \langle x_a x_b\rangle\langle x_c x_d\rangle + \langle x_a x_c\rangle\langle x_b x_d\rangle + \langle x_a x_d\rangle\langle x_b x_c\rangle,$$

an instance of Wick's theorem

c) Show that

$$\langle e^{ix_a}\rangle = e^{-\frac{1}{2}\langle x_a^2\rangle}$$