Remarks on Problem 14

1. Dezember 2016

For Problem 14 you were asked to minimize the Landau free energy $\mathcal{F}$ with respect to both $\tilde{M}_n$ *and* n. While technically correct it is leaving out a few intermediate steps that I will sketch below.

As the Fourier modes in $\mathcal{F}$ are not coupled, the partition function reads

$$Z = \prod_n \int d\tilde{M}_n e^{-\beta \mathcal{F}_n[\tilde{M}_n]} \tag{1}$$

where $\mathcal{F} = \sum_n \mathcal{F}_n$.

Now one should minimize the Landau free energy for every $\tilde{M}_n$ individually. For $\kappa < 0$ one will find a transition temperature $t_n(\kappa) \neq 0$ where the optimal $\overline{M}_n$ becomes non-zero. In general, the t_n will all be different. If we are now only interested in the transition from a homogeneous to a spatially modulated phase, we are looking for the largest t_n. The Fourier mode $\overline{M}_n$ that lowers the free energy the most for $n \neq 0$ will be the first that will become non-zero and it will have the highest transition temperature t_n. Therefore, it will be at a minimum of $\mathcal{F}_n[\overline{M}_n]$ with respect to n (or q_n). Although we minimze for both variables, it is for different reasons.

Now, technically, and with these considerations in mind it is even easier to minimize $\mathcal{F}_n$ with respect to q_n first and only then minimize $\mathcal{F}_{\overline{n}}[\tilde{M}_{\overline{n}}]$ with respect to $\tilde{M}_{\overline{n}}$ for the optimal $\overline{n}$ which was the approach I had handed out to the tutors but which is slightly backwards in light of the above.